

Complex Analysis

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Chapter 0

Introduction

BLOCKS OF THE COURSE:

- Measure (measurable functions) and integral (integrable function)
- MTC, DCT

Example. SUPER IMPORTANT EXAMPLE!! Let $f_n(x) = n^2 x^n (1 - x)$ ($0 \leq x \leq 1$). Then $\lim_{n \rightarrow \infty} f_n(x) = 0$ for all $x \in [0, 1]$, but $\lim_{n \rightarrow \infty} \int_0^1 f_n(x) dx = 1$. This example arises in any reasonable theory.

We want theorems of the form (f_n) is a sequence of integrable functions, $f_n(x) \rightarrow f(x)$ for each x , and [additional assumptions]. Then f is integrable and $\int f(x) dx = \lim_{n \rightarrow \infty} \int f_n(x) dx$.

Monotone Convergence Theorem, Dominated Convergence Theorem.

Crucial ideas for Lebesgue's construction:

- Instead of using integrals to define lengths of sets, define length of a set directly; then define integrals.
- Instead of partitioning the x -axis into intervals and using step functions, partition the y -axis into intervals and consider corresponding "simple" functions.